REPORT DOCUMENTATION PAGE AD-A260 174

Form Approved
OMB No. 0704-0188

2

1. AGENCY USE ONLY (Leave blank)

2. REPORT DATE Jan 93

3. REPORT TYPE AND DATES COVERED Technical

4. TITLE AND SUBTITLE
RESEARCH IN STOCHASTIC PROCESSES
AND THEIR APPLICATIONS

5. FUNDING NUMBERS
DAAL03-92-G-0008

6. AUTHOR(S)
Cambanis, S.
Kallianpur, G.
Leadbetter, M.R.

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Statistics Dept.
UNC- Chapel Hill
Chapel Hill, NC 27599-3260

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
U. S. Army Research Office
P. O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
ARO 29297.1-MA

11. SUPPLEMENTARY NOTES
The view, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

Research was conducted and directed in the area of stochastic processes and their applications in engineering, neurophysiology and oceanography by the principal investigators, S. Cambanis, G. Kallianpur and M.R. Leadbetter and their associates.

14. SUBJECT TERMS
(over)

15. NUMBER OF PAGES 52

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

14. Subject terms.

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Stochastic differential equation models for spatially distributed neurons

Propagation of chaos for interacting systems

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Wavelets, multiresolution decomposition, and random processes

Non-Gaussian stable models: Structure and inference

Inference for linear and harmonizable time series

Periodically correlated and other nonstationary processes

Sample function properties

Random fields and their prediction

Markov random field models for vision

Point processes, random sets, and random measures

Random measures associated with high levels

Tail inference for stationary sequences

RESEARCH IN STOCHASTIC PROCESSES
AND THEIR APPLICATIONS

Co-Principal Investigators:
Professor Stamatis Cambanis
Professor Gopinath Kallianpur
Professor M. Ross Leadbetter

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Unannounced	☐
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Availability Codes	
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A-1	

Department of Statistics
University of North Carolina
Chapel Hill, NC 27599-3260

Army Research Office Grant No. DAAL 03 92 G 0008

ANNUAL SCIENTIFIC REPORT

Period: 1 January 1992 through 30 September 1992

RESEARCH IN STOCHASTIC PROCESSES AND THEIR APPLICATIONS

CONTENTS

RESEARCH IN STOCHASTIC PROCESSES AND THEIR APPLICATIONS

SUMMARY OF RESEARCH ACTIVITY

Research was conducted and directed in the area of stochastic processes and their applications in engineering, neurophysiology and oceanography by the principal investigators, S. Cambanis, G. Kallianpur and M.R. Leadbetter and their associates. A list of the main areas of research activity follows. More detailed descriptions of the work of all participants is given in the main body of the report.

Stochastic differential equations in infinite dimensional spaces

Stochastic differential equation models for spatially distributed neurons

Propagation of chaos for interacting systems

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RESEARCH IN STOCHASTIC PROCESSES AND THEIR APPLICATIONS

STAMATIS CAMBANIS

The work briefly described here was developed in connection with problems arising from and related to the statistical communication theory and the analysis of stochastic signals and systems.

Part I considers questions raised by the observation of continuous time random signals at discrete sampling times, and the transmission or storage of analog random signals in digital form.

Part II considers non-Gaussian models frequently encountered in practical applications. The goal is to learn how Gaussian and linear signal processing methodologies should be adapted to deal with non-Gaussian regimes.

Part III continues the study of wavelets and multiresolution analysis for random processes, and Part IV deals with random filtering and the harmonic analysis of nonstationary processes.

Item 5 is continuing joint work with E. Masry of the University of California, San Diego. Items 3, 4, 6, and 8 are in collaboration with visitors to the Center for Stochastic Processes: Houdré, Hurd, Fakhre-Zakeri, Leskow, Mandrekar, Rosinski and Surgailis. Items 1 and 2 are continuing work with former Ph.D. students Benhenni and Su.

I. DIGITAL PROCESSING OF ANALOG SIGNALS

Continuous time signals are typically sampled at discrete times and inferences are made on the basis of these samples, which may be further quantized (or rounded-off) for digital processing. Items 1 and 2 describe work in progress on sampling designs for the estimation of regression coefficients and on the degradation of the performance of sampling designs due to quantization.

1. Sampling designs for regression coefficient estimation with correlated errors. [1]

The problem of estimating regression coefficients from observations at a finite number of properly designed sampling points is considered when the error process has correlated values. Sacks and Ylvisaker (1966) found an asymptotically optimal design for the best linear unbiased estimator, which generally may lack numerical stability and requires the precise knowledge of the covariance function of the error process. Su and Cambanis (1991) found an asymptotically optimal design for a simpler estimator which is relatively nonparametric (with respect to the error covariance function) when the error has no quadratic mean derivative. This was achieved by properly adjusting the median sampling design and the simpler estimator introduced by Schoenfelder (1978). Here simpler yet sampling designs and estimators are introduced which have asymptotically optimal performance even for smoother error processes (with quadratic mean derivatives).

2. The effect of quantization on the performance of sampling designs. [2]

The most common form of quantization is rounding-off, which occurs in all digital systems. A general quantizer approximates an observed value by the nearest among a finite number of representative values. In estimating weighted integrals of time series with no quadratic mean derivatives, by means of samples at discrete times it is known that the rate of convergence of the mean square error is reduced from n^{-2} to $n^{-1.5}$ when the samples are quantized (Bucklew and Cambanis (1988)). For smoother time series, with $k = 1, 2, \ldots$ quadratic mean derivatives, it is now shown that the rate of convergence is reduced from n^{-2k-2} to n^{-2} when the samples are quantized, which

is a very significant reduction. The interplay between sampling and quantization is also studied, leading to (asymptotically) optimal allocation between the number of samples and the number of levels of quantization.

II. NON-GAUSSIAN MODELS

In continuing the exploration of non-Gaussian models we have studied a couple of stable models. A new rich class of stationary stable processes generalizing moving averages is introduced and studied in Item 3, and the linearity property of the prediction of heavy-tailed autoregressive processes in reversed time is characterized in Item 4.

3. Generalized stable moving averages. [3]

No explicit representation is known for all stationary non-Gaussian stable processes. The main two subclasses studied, which have explicit representations motivated by the Gaussian case, are the harmonizable processes, which are superpositions of harmonics with stable amplitudes, and the moving average processes, which are filtered white stable noise. While in the Gaussian case, the latter is a subclass of the former, in the non-Gaussian stable case the two classes are disjoint. The study of stable moving average processes is facilitated by the fact that their distribution is essentially (except for a translation and sign) determined by the filter function of the moving average. This has made it possible to study distributional properties of the process (mixing, ergodicity, self-similarity, Markov property, etc.) through the properties of the filter functions.

In this work the class of non-Gaussian stable moving average processes is expanded substantially by the introduction of an appropriate joint randomization of the filter function and of the stable noise, leading to stable generalized moving averages (GMA). The characterization of their distribution through their filter function and their mixing measure leads to a far reaching generalization of a theorem of Kanter (1972).

It is shown that stable GMA's contain sums of independent stable moving averages and that they are still disjoint from the harmonizable processes, but are closed under time invariant filters, and that they are mixing, so they have strong ergodic properties. They lead to a wealth of new examples of self-similar processes, beyond the linear fractional stable motions, and also of processes which are reflection positive, which is a useful weakening of the Markov property.

4. The prediction of heavy-tailed autoregressive sequences: Regression versus best linear prediction. [4]

The prediction of heavy-tailed first order autoregressive sequences is considered. In forward time the regression on all past values is the same as the one-step regression on the previous value, which is in fact linear. In reversed time the regression on all future values is the same as the one-step regression on the immediate future value (i.e. the Markovian property is retained) and we show that it is linear if and only if the innovations have a semistable distribution. This answers a question posed by Rosenblatt (1992) who considered sequences with finite second moment and showed that regression with time reversed is linear if and only if the innovations are Gaussian.

When the distribution of the innovations is non-Gaussian stable, then both regressions in forward and reversed time are linear, but while the forward regression is the best linear predictor, the regression with time reversed is not! The performance of linear regression predictor is compared in this case with that of the best linear predictor.

III. MULTIRESOLUTION DECOMPOSITION AND WAVELET TRANSFORMS OF RANDOM SIGNALS

The wavelet approximation of deterministic and random signals at given resolution is considered in Item 5, which is a substantial generalization of earlier work and encompasses a much larger class of wavelets. The properties of the wavelet transform of random signals are considered in the nearly completed work in Item 6. Further studies are currently under way.

5. Wavelet approximation of deterministic and random signals: Convergence properties and rates. [5]

An n^{th} order asymptotic expansion is developed for the error in the wavelet approximation at resolution 2^{-k} of deterministic and of random signals. The deterministic signals are assumed to have n continuous derivatives, while the random signals are only assumed to have a correlation function with continuous n^{th} order derivatives off the diagonal - a very mild assumption. For deterministic signals over the entire real line, for stationary random signals over finite intervals, and for nonstationary random signals with finite mean energy over the entire real line, the moments of the scale function can be matched with the signal smoothness to improve substantially the quality of the approximation. In sharp contrast this does not appear to be generally feasible for nonstationary random signals over finite intervals, as well as for deterministic signals which are only locally square integrable.

6. Wavelet transforms of random processes. [6]

A study has been initiated of the properties of wavelet transforms of random processes whose sampled values appear as coefficients in the wavelet approximation of the process at a given resolution. A natural question is which properties of the process are inherited to its wavelet transform, and, conversely, which properties of the process can be read-off properties of its wavelet transform. For random processes with finite second moment, properties such as periodicity, stationarity, harmonizability, and self-similarity, are characterized by means of analogous properties of their wavelet transforms at some scale: The properties of the wavelet transform characterize the corresponding properties of the increments of the process of order equal to the order of regularity of the analyzing wavelet.

IV. NON-STATIONARY PROCESSES

In pursuing the study of non-stationary processes, the random filters which preserve the normality of certain non-stationary random inputs are characterized in Item 7, and further classes of non-stationary inputs are currently under study; earlier work on weak laws of large numbers for periodically and for almost periodically correlated processes which are not stationary or harmonizable was substantially revised; and work is in progress jointly with A.G. Miamee of Hampton University on continuous-time correlation-autoregressive sequences.

7. Random filters which preserve the normality of non-stationary random inputs. [7]

When a Gaussian signal goes through a non-random linear filter, its output is also Gaussian. We are interested in characterising and identifying those random linear filters which are independently distributed of their random inputs and preserve their normality. If the input is a stationary Gaussian process, then the output is Gaussian only when the linear filter has non-random gain. Here we consider non-stationary random inputs, for which the situation is more delicate. When the input

has stationary independent Gaussian increments, then the output is Gaussian only for linear filters with either non-random gain or random sign! On the other hand when the Gaussian input has non-stationary independent bounded increments, or is a non-stationary bounded noise (possibly dependent), or is harmonizable with diffused spectral measure, then the output is Gaussian only for linear filters with random sign. The non-random characteristics of these filters can be identified from the Gaussian distributions of the input and output processes, and their random characteristics from the joint distribution of input the output, which cannot be Gaussian unless the filter is non-random.

8. Laws of large numbers for periodically and almost periodically correlated processes. [8]

This paper gives results related to and including laws of large numbers for (possibly non-harmonizable) periodically and almost periodically correlated processes. These results admit periodically correlated processes that are not continuous in quadratic mean. The idea of a stationarizing random shift is used to show that strong law results for weakly stationary processes may be used to obtain strong law results for such processes.

A substantial revision of this work from last year is being completed. Important examples have been added and the development for almost periodically processes has been made simpler and more transparent.

References

[1] K. Benhenni, S. Cambanis and Y.C. Su, Sampling designs for regression coefficient estimation with correlated errors, in preparation

[2] K. Benheni and S. Cambanis, The effect of quantization on the performance of sampling designs, in preparation

[3] S. Cambanis, V. Mandrekar, J. Rosinski and D. Surgailis, Generalized stable moving averages, UNC Center for Stochastic Processes Technical Report No. 365, April 1992

[4] S. Cambanis and I. Fakhre-Zakeri, On prediction of heavy-tailed autoregressive sequences: Regression versus best linear prediction, UNC Center for Stochastic Processes Technical Report No. 383, December 1992

[5] S. Cambanis and E. Masry, Wavelet approximation of deterministic and random signals: Convergence properties and rates, UNC Center for Stohcastic Processes Technical Report No. 352, Nov. 1991, revision in preparation

[6] S. Cambanis and C. Houdré, Wavelet transforms of random processes, in preparation

[7] S. Cambanis, Random filters which preserve the normality of non-stationary random inputs, Nonstationary Stochastic Processes and their Applications, A.G. Miamee ed., World Scientific, 1992, 219-237

[8] S. Cambanis, C. Houdré, H.L. Hurd and J. Leskow, Laws of large numbers for periodically and almost periodically correlated processes, UNC Center for Stochastic Processes Technical Report No. 334, Mar. 1991, revision in preparation

GOPINATH KALLIANPUR

As in recent years, the major areas of my research have been the following:

I. Stochastic differential equations in infinite dimensional spaces

II. Nonlinear white noise analysis

III. Feynman integrals and integration in Hilbert space

IV. Prediction theory of second order stationary random fields

A description of the research done under each heading is given below:

I. STOCHASTIC DIFFERENTIAL EQUATIONS (SDE's) IN INFINITE DIMENSIONAL SPACES

The continuing research in this area is an attempt to develop a theory of infinite dimensional dynamical systems. A major emphasis of the present work is on investigating SDE's in duals of nuclear spaces driven by discontinuous noise sources, in particular, Poisson random measures. Most of the existing theory is devoted to SDE's or stochastic partial differential equations (SPDE) driven by cylindrical Brownian motions or space-time Wiener processes primarily because of its mathematical elegance and the link with diffusion processes.

However, in problems of neuronal behavior, environmental pollution and fluid mechanics (to name only a few fields of application) it seems more natural to consider dynamic models, i.e. SDE's or SPDE's driven by Poisson random measures. The diffusion approximations that can be derived from them throw additional light on the continuous models. The new work (jointly with J. Xiong) described below pertains to recent work on the application to environmental pollution [3], uses the techniques and results of the following papers partially described in the Annual Scientific Report for 1990-1991 and is now completed:

1. The existence and uniqueness of the solution of nuclear space-valued stochastic differential equations driven by Poisson random measures (with G. Hardy, S. Ramasubramanian and J. Xiong) [1]

In this paper, we study SDE's on duals of nuclear spaces driven by Poisson random measures. The existence of a weak solution is obtained by the Galerkin method. For uniqueness, a class of ℓ^2-valued processes which are called Good processes is introduced. An equivalence relation is established between SDE's driven by Poisson random measures and those by Good processes. The uniqueness is established by extending the Yamada-Watanabe argument to the SDE's driven by Good processes. This is an extension to discontinuous infinite dimensional SDE's of work done by G. Kallianpur, I. Mitoma and R. Wolpert for nuclear space valued diffusions [Stochastics. 29, 1-45, (1990)].

2. Asymptotic behavior of a system of interacting stochastic differential equations driven by Poisson random measures (with J. Xiong) [2]

In this paper, we study a system of interacting stochastic differential equations taking values in nuclear spaces and driven by Poisson random measures. We also consider the McKean-Vlasov equation associated with the system. We show that under suitable conditions the system has a unique solution and the sequence of its empirical distributions converges to the solution of the McKean-Vlasov equation when the size of the system tends to infinity. The results are applied to the voltage potentials of

a large system of neurons and a law of large numbers for the empirical measure is obtained.

3. Stochastic models of environmental pollution (with J. Xiong) [3]

In this paper, we consider several stochastic models arising from environmental problems. First, we study the pollution in a domain where undesired chemicals are deposited at random times and locations according to Poisson streams. Incorporated with drift and dispersion, the chemical concentration can be modeled by linear stochastic partial differential equations (SPDE) which are solved by applying the general results of Kallianpur and Xiong (SDE's in infinite dimensions: A brief survey and some new directions, Center for Stochastic Processes Technical Report No. 372, Sept. 92).

We examine in a somewhat more general context, the stochastic dynamic model considered by Kwakernaak and by Curtain, and look at the problem in the framework of general SPDE's: Let

$$x \in \mathcal{X} = [0, \ell]^d, \quad x = (x_1, \ldots, x_d).$$

The underlying deterministic PDE is

$$\frac{\partial u}{\partial t} = D\Delta u - V. \nabla u + \alpha u, \quad t > 0,$$

where $u = u_t(x)$, $D > 0$, $V = (V_1, \ldots V_d)$ and α are constants,

$$\Delta = d - \text{dimensional Laplacian and} \quad \nabla = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \ldots, \frac{\partial}{\partial x_d}\right).$$

Let

$$c_i = \frac{V_i}{2D_i}, \ \rho(x) = e^{-2(c_1 x_1 + \ldots + c_d x_d)} \quad \text{and} \quad H_0 = L^2(\mathcal{X}, \rho(x)dx).$$

The cases $d = 1, 2$ or 3 are of physical interest,

$$d = 1 \quad (\text{River Pollution}), \quad d = 2 \ (or 3) \ (\text{Atmospheric Pollution}).$$

We impose the (Neumann) boundary conditions (for $d = 2$),

$$\frac{\partial}{\partial x_1} u_t(0, x_2) = \frac{\partial}{\partial x_1} u_t(\ell, x_2) = \frac{\partial}{\partial x_2} u_t(x_1, 0) = \frac{\partial}{\partial x_2} u_t(x_1, \ell) = 0 \quad (t > 0).$$

The problem defines a positive, self-adjoint operator denoted by $-L$ on H_0 and (T_t) is the semigroup generated by L.

Let $\lambda_0^i = 0$, $\lambda_j^i = D(c_i^2 + \frac{j^2\pi^2}{\ell^2})$, $i = 1, \ldots, d$.

$$\phi_0^i(y) = (\frac{2c_i}{1 - e^{-2c_i \ell}})^{1/2}, \quad \phi_j^i(y) = (\frac{2}{\ell})^{1/2} e^{c_i y} \sin(\frac{j\pi}{\ell} y + \alpha_j^i),$$

where $\alpha_j^i = \tan^{-1}(-\frac{j\pi}{\ell c_i})$, $j \geq 1$. Then $\lambda_{j_1, \ldots, j_d}$ and $\phi_{j_1, \ldots, j_d}(x)$, where

$$\lambda_{j_1, \ldots, j_d} = \lambda_{j_1}^1 + \ldots + \lambda_{j_d}^d, \quad \phi_{j_1 \ldots j_d}(x) = \phi_{j_1}(x_1) \ldots \phi_{j_d}(x_d),$$

are the eigenvalues and eigenfunctions of $-L$. $\{\phi_{j_1 \ldots j_d}\}$ is a CONS in H_0. With the help of these we define a linear space $\Phi \subseteq H_0$ of smooth functions in H_0 and we have a chain

$$\Phi \subset \ldots \subset H_p \subset \ldots \subset H_0 \subset H_{-1} \subset \ldots \subseteq H_{-p} \subseteq \ldots \subseteq \Phi'.$$

$\Phi = \cap_p H_p$; $H_{-p} = H_p^*$, H_p (Hilbert space). Φ is a Fréchet space which is also nuclear.

The natural assumption is that the influx of pollution follows a Poisson process. More precisely, let $\mathcal{N}$ be a Poisson random measure on $\mathbf{R}_+ \times \mathcal{X} \times \mathbf{R}_+$ ($\mathbf{R}_+ = (0, \infty)$) with $E\mathcal{N}(dt\ dx\ da) = dt\mu(dx\ da)$ where μ is the intensity measure on $\mathcal{X} \times \mathbf{R}_+$. (At a random time τ_j, there is a random accretion A_j of pollution).

Then the stochastic version of the above equation is given by

$$u_t[\varphi] = u_0[\varphi] + \int_0^t A(s, u_s)[\varphi]ds + \int_0^t \int_{\mathcal{X}} \int_{\mathbf{R}_+} a\varphi(x)\tilde{\mathcal{N}}(ds\ dx\ da)$$

($\tilde{\mathcal{N}}$ = compensated random measure),

$$A(s, v)[\varphi] = -v[L\varphi] + \alpha v[\varphi] + \int_{\mathcal{X}} \int_{\mathbf{R}_+} a\varphi(x)\mu(dxda).$$

We have the following result which shows that, in general, the SDE has a solution in $L^2[0, \ell]^d$ so that we do not have to seek a distributional solution.

Theorem 2 Suppose $E \parallel u_0 \parallel_{H_0}^2 < \infty$ and μ is a finite measure. Let

$$\left| \int_{\mathcal{X}} \int_{\mathbf{R}_+} a\varphi(x)\mu(dx\ da)\right|^2 + \int_{\mathcal{X}} \int_{\mathbf{R}_+} a^2\varphi(x)^2(dx\ da) \ \leq \ \text{Const.} \parallel \varphi \parallel_0^2 .$$

Then

$$u_t \in H_0 = L^2([0, \ell]^d, \rho dx) \ \text{ a.s. } \forall t.$$

For $d = 1$, $E \parallel u_t \parallel_0^2 < \infty$. For $d > 1$, $E \parallel u_t \parallel_0^2 = \infty\ \forall t$ in the most interesting cases.

Two more realistic models of pollution are studied (d=1):

(i) Pollution emission at specific sites (Factories);

(ii) Pollution model with an upper tolerance level for chemical concentration.

(i) Suppose there are r sites $K_1, \ldots, K_r$ at which chemicals are deposited in terms of independent Poisson streams $\mathcal{N}_i(t)$ with parameter $f_i > 0$, with random magnitudes $\{A_i^j\}(j = 1, 2, \ldots)$ where A_i^j have common d.f. $F_i(da)$ arriving in the vicinity $(K_i - \epsilon_i, K_i + \epsilon_i)$ of K_i. For a set $A \subset [0, \ell]$ and $B \subset \mathbf{R}_+$ let

$$\mathcal{N}((0, t] \times A \times B) = \sum_{i=1}^{r} 1_A(K_i) \sum_{j=1}^{\mathcal{N}_j(t)} 1_B(A_i^j)$$

with

$$\mu(A \times B) = \sum_{i=1}^{r} 1_A(K_i) f_i F_i(B).$$

The new SDE takes the form

$$u_t[\varphi] = u_0[\varphi]ds + \int_0^t A(s, u_s)[\varphi]ds + \int_0^t \int_{\mathcal{X}} \int_{\mathbf{R}_+} G(s, u_{s-}, x, a)\tilde{\mathcal{N}}(ds\ dx\ da)$$

where

$$
\begin{aligned}
G(t,v,(x,a))[\varphi] &= \frac{a}{2\epsilon_i}\int_{K_i-\epsilon_i}^{K_i+\epsilon_i}\phi(y)dy \quad \text{if} \quad x = K_i,\ i=1,\ldots,r, \\
&= 0 \text{ otherwise}, \\
A(t,v)[\varphi] &= -v[L\varphi] + \alpha v[\varphi] + m[\varphi], \\
m[\varphi] &= \sum_{i=1}^{r}\frac{f_i a_i}{2\epsilon_i}\int_{K_i-\epsilon_i}^{K_i+\epsilon_i}\phi(y)dy
\end{aligned}
$$

and

$$a_i = \int_{\mathbf{R}_+} aF_i(da).$$

(ii) We consider a simple model where the upper level for pollution is a fixed tolerance function $\xi(x)$. Assume that the change of chemical concentration does not depend on the locations where the polluted material is deposited. We then have a quasilinear SDE

$$u_t[\varphi] = u_0[\varphi] + \int_0^t \{u_s[-L\varphi] + \alpha u_s[\varphi]\}ds + \int_0^t \int_{\mathbf{R}_+} a(\xi[\varphi] - u_{s-}[\varphi])N(ds\ da).$$

N being a Poisson random measure with intensity measure μ on $\mathbf{R}_+$.

If the initial value u_0 is smooth and $\alpha \leq 0$, it is shown that the above quasi-linear SDE has a solution in $D([0,T],\Phi)$ and furthermore, under suitable conditions that the total amount of pollution cannot exceed a prescribed bound.

Finally, the asymptotic behavior as $t \to \infty$ of the solution of the SDE in (i) is investigated and the following diffusion approximation is obtained:

$$\frac{\partial \tilde{u}_t}{\partial t} = D\Delta\tilde{u}_t - V \cdot \nabla\tilde{u}_t + \alpha\tilde{u}_t + \dot{W}_{tx}$$

where W_{tx} is space-time Gaussian white noise

$$W_{tx} = \sum_{j=1}^{r} b_j^{1/2}\delta_{K_j}(x)e^{-cx}B_j(t),$$

where $B_j(t)$ are independent real Brownian motions, $b_j = \int_0^\infty a^2 F_j(da)$ and δ_K is Dirac measure at K.

Research on interacting Hilbert space valued diffusions carried out in collaboration with A. Bhatt and R.L. Karandikar (Annual Scientific Report 1990-1991) has provided results that have been applied to the asymptotic behavior of interacting neurons in the following paper:

4. Stochastic differential equation models for spatially distributed neurons and propagation of chaos for interacting systems [4]

Distribution or nuclear space valued SDE's (diffusions as well as discontinuous equations) are discussed as stochastic models for the behavior of voltage potentials of spatially distributed neurons. A propagation of chaos result is obtained for an interacting system of Hilbert space valued SDE's.

II. APPLICATIONS OF NONLINEAR WHITE NOISE STOCHASTIC ANALYSIS

The question of when a nonlinear transformation of the Wiener measure μ is absolutely continuous with respect to μ is a difficult problem that has been outstanding since the time of Cameron and Martin who were the first to investigate it. The most important work since then has been done by R. Ramer and generalized by S. Kusuoka.

A nonlinear theory of white noise on Hilbert space developed by R.L. Karandikar and myself has provided a new way to approach this problem. This research is presented in the following paper:

5. Nonlinear transformations of the canonical Gauss measure on Hilbert space and absolute continuity (with R.L. Karandikar) [5]

The papers of R. Ramer (1974) and S. Kusuoka (1982) investigate conditions under which the probability measure induced by a nonlinear transformation on abstract Wiener space (γ, H, B) is absolutely continuous with respect to the abstract Wiener measure μ. These conditions reveal the importance of the underlying Hilbert space H but involve the space B in an essential way. The present paper gives conditions solely based on H and takes as its starting point a nonlinear transformation $T = I + F$ on H. New sufficient conditions for absolute continuity are given which do not seem easily comparable with those of Kusuoka or Ramer but are more general than those of Buckdahn (1991) and Enchev (1991). The Ramer-Itô integral occurring in the expression for the Radon-Nikodym derivative is studied in some detail and, in the general context of white noise theory, it is shown to be an anticipative stochastic integral which, under a stronger condition on the weak Gateaux derivative of F, is directly related to the Ogawa integral.

III. FEYNMAN INTEGRALS: FUNCTIONAL INTEGRALS OVER HILBERT SPACES RELATED TO THE FEYNMAN INTEGRAL.

6. Integration over Hilbert spaces: Examples inspired by the harmonic oscillator (with V. Papanicolaou) [6]

The work is joint with V. Papanicolaou and is briefly described under his heading.

IV. PREDICTION THEORY OF SECOND ORDER STATIONARY RANDOM FIELDS

7. Spectral characterization and autoregressive expansion of linear predictors for second order stationary random fields (SOSRF), Part I (with J. Farshidi and V. Mandrekar) [7]

The problem of finding spectral criteria for autoregressive (AR) expansions is of great practical importance for single parameter stationary time series. While this problem has been solved satisfactorily in recent years, the corresponding problem for SOSRF has only now come to the forefront.

The major difficulty with SOSRF is that there is no unique definition of "past" and "future". The definitions of a deterministic and purely nondeterministic random field can therefore be given separately for the horizontal, vertical and "south west" or quarter plane past.

An AR expansion is an expansion for the linear least squares predictor given in terms of past **observations** rather than in terms of an innovation sequence based on the past. The original solution of the Kolmogorov-Wiener theory is based on the

latter and is less easily used in practiced than an AR expansion. A paper is being prepared on the work done to date.

References

[1] G. Hardy, G. Kallianpur, S. Ramasubramanian and J. Xiong, The existence and uniqueness of solutions of nuclear space-valued stochastic differential equations driven by Poisson random measures, UNC Center for Stochastic Processes Technical Report No. 348, June 92

[2] G. Kallianpur and J. Xiong, Asymptotic behavior of a system of interacting stochastic differential equations driven by Poisson random measures, in preparation

[3] G. Kallianpur and J. Xiong, Stochastic models of environmental pollution, in preparation

[4] G. Kallianpur, Stochastic differential equation models for spatially distributed neurons and propagation of chaos for interacting systems, *J. Math. Biol.*, to appear.

[5] G. Kallianpur and R.L. Karandikar, Nonlinear transformations of the canonical Gauss measure on Hilbert space and absolute continuity, in preparation

[6] G. Kallianpur and V. Papanicolaou, Integration over Hilbert spaces: Examples inspired by the Harmonic Oscillator, UNC Center for Stochastic Processes Technical Report No. 367, July 92

[7] J. Farshidi, G. Kallianpur and V. Mandrekar, Spectral characterization and autoregressive expansion of linear predictors for second order stationary random fields, Part I, UNC Center for Stochastic Processes Technical Report No. 381, Dec. 92

M. ROSS LEADBETTER

Together with S. Cambanis and G. Kallianpur, M.R. Leadbetter provided continuing direction and participation in the research activities of the Center for Stochastic Processes. Since this was at no contract cost during the current year, a brief activity summary is given here rather than a detailed contract reporting. The activities are described by area as follows

1. Tail inference for stochastic sequences.

Work previously reported (cf. Center for Stochastic Processes Technical Report No. 292) was further developed jointly with H. Rootzén. This concerns problems such as estimation of parameters of exponentially or regularly varying tail distributions, extremal index, tail probabilities and quantiles. This work, originally planned for a single publication, is now being expanded into two parts.

2. Convergence of vector random measures.

Research with S. Nandagopalan on convergence of vector random measures was developed in the current report period and will be completed in the subsequent months. General theorems are given for convergence, with particular reference to the random measure formed from multilevel exceedances by a (nonstationary) stochastic process.

3. Processes with deterministic peaks.

Stationary Gaussian processes have the property that high peaks have an increasingly parabolic asymptotic form. This notion can be generalized leading to the concept of "deterministic peaks" – where the time above a high level (asymptotically) determines that above any higher level. This ongoing work will be described shortly in a paper (joint with T. Hsing).

4. Excursion random measures.

Extreme value behavior of stochastic sequences can be summarized by limiting behavior of the two dimensional point process formed by plotting (a suitably normalized version of) the sequence in the plane. Substantial effort - jointly with T. Hsing - has been put into the development of a corresponding continuous time theory in this and previous reporting periods (cf. Center for Stochastic Processes Technical Report No. 350). This work is now undergoing revision for publication.

5. Applications.

Work (jointly with a student, L.S. Huang) was initiated on the application of "exceedance methods" to environmental data. Some preliminary time series modeling of ozone data has been undertaken and will be continued in the coming year.

RAY CHENG

Professor Ray Cheng of the Department of Mathematics of the University of Louisville visited the Center for two months and worked on the structure of two-parameter random fields which is relevant to the problem of prediction. He completed the following technical reports.

1. Outer factorization of operator valued weight functions on the torus [1]

An exact criterion is derived for an operator-valued weight function $W(e^{is}, e^{it})$ on the torus to have a factorization $W(e^{is}, e^{it}) = \Phi(e^{is}, e^{it})^{*}\Phi(e^{is}, e^{it})$, where the operator valued Fourier coefficients of Φ vanish outside of the Helson-Lowdenslager halfplane $\Lambda = \{(m, n) \in \mathbf{Z}^2 : m \geq 1\} \cup \{(0, n) : n \geq 0\}$, and Φ is "outer" in a related sense. The criterion is expressed in terms of a regularity condition on the weighted space $L^2(W)$ of vector valued functions on the torus. A logarithmic integrability test is also provided. The factor Φ is explicitly constructed in terms of Toeplitz operators and other structures associated with W. The corresponding version of Szegö's infimum is given.

2. Operator valued functions of several variables: Factorization and invariant subspaces [2]

This work is an attempt to extend the classical function theory on the Hardy space H^2 to certain classes of operator valued functions of several variables. Of course, it is impossible to carry over all of the interesting details. Our focus is to adapt the notions of inner and outer functions, so as to preserve two basic factorization theorems. We also establish a sort of Beurling-Lax theorem to describe a class of associated invariant subspaces. The overall approach concerns functions on the torus, which generally cannot be realized as the boundary limits of analytic functions in the complex sense. Accordingly, our techniques are chiefly borrowed from multiple Fourier series and shift analysis.

References

[1] R. Cheng, Outer factorization of operator valued weight functions on the torus, UNC Center for Stochastic Processes Technical Report No. 371, July 92

[2] R. Cheng, Operator valued functions of several variables: Factorization and invariant subspaces, UNC Center for Stochastic Processes Technical Report No. 379, Nov. 92

ISSA FAKHRE-ZAKERI

Professor Issa Fakhre-Zakeri of the Department of Mathematics of the University of Maryland visited the cetner during the 1992 year. He worked on inference problems for stationary linear time series with finite variance jointly with J. Farshidi [1,2] and with heavy tails jointly with S. Cambanis [3].

1. A central limit theorem with random indices for stationary linear processes [1]

A central limit theorem with random indices is obtained for stationary linear process $X_t - \mu = \sum_{j=0}^{\infty} a_j \eta_{t-j}$, where $\{\eta_t\}$ are independent and identically distributed random variables with mean zero and finite variance and $\sum_{j=0}^{\infty} |a_j| < \infty$.

2. Limit theorems for sample covariances of stationary linear processes with applications to sequential estimation [2]

For a stationary linear process the strong consistency and rate of convergence are established under optimal conditions for the asymptotic variance of their sample mean. Applications are made to the problem of sequential point and fixed width confidence interval estimation of the mean of a stationary linear process.

3. On prediction of heavy-tailed autoregressive sequences: Regression versus best linear prediction [3]

The prediction of heavy-tailed first order autoregressive sequences is considered. In forward time the regression on all past values is the same as the one-step regression on the previous value, which is in fact linear. In reversed time the regression on all future values is the same as the one-step regression on the immediate future value (i.e. the Markovian property is retained) and we show that it is linear if and only if the innovations have a semistable distribution. This answers a question posed by Rosenblatt (1992) who considered sequences with finite second moment and showed that regression with time reversed is linear if and only if the innovations are Gaussian.

When the distribution of the innovations is non-Gaussian stable, then both regressions in forward and reversed time are linear, but while the forward regression is the best linear predictor, the regression with time reversed is not! The performance of linear regression predictor is compared in this case with that of the best linear predictor.

References

[1] I. Fakhre-Zakeri and J. Farshidi, A central limit theorem with random indices for stationary linear processes. UNC Center for Stochastic Processes Technical Report No. 363, April 1992

[2] I. Fakhre-Zakeri and J. Farshidi. Limit theorems for sample covariances of stationary linear processes with applications to sequential estimation. UNC Center for Stochastic Processes Technical Report No. 380, September 1992

[3] S. Cambanis and I. Fakhre-Zakeri, On prediction of heavy-tailed autoregressive sequences: Regression versus best linear prediction. UNC Center for Stochastic Processes Technical Report No. 383, December 1992

JAMSHID FARSHIDI

Dr. Jamshid Farshidi from the Department of Probability and Statistics of Michigan State Unviersity spent the academic year as a postdoctoral visitor to the center. He worked on the problem of prediction of stationary time series [1] and of random fields [2] and on inference for stationary linear time series (jointly with I. Fakhre-Zakeri) [3,4]. He has also begun working on heavy tailed stationary time series and more specifically on the prediction of harmonizable stationary stable processes.

1. Autoregressive expansion of the linear predictor for stationary stochastic processes [1]

The principal problems considered are the existence and uniqueness of an autoregressive expansion of the linear predictor for a discrete stationary process with spectral density f and optimal factor φ, and the invertibility of the process X. The main results are:

(1) the equivalence of the strong convergence of an autoregresive series to the linear predictor, with its boundedness, and with its weak convergence;

(2) the uniqueness of an autoregressive expansion;

(3) the equivalence of an autoregressive expansion with the invertibility of the process;

(4) the sufficiency of the condition $(1/f) \in L^1$ for the existence, convergence, uniqueness of the autoregressive expansion and the invertibility of the process;

(5) a necessary condition based on φ and f for the existence, uniqueness, and convergence of an autoregressive expansion, and invertibility of the process.

2. Spectral characterization and autoregressive expansion of linear predictors for second order stationary random fields (SOSRF), Part I (with G. Kallianpur and V. Mandrekar) [2]

The problem of finding spectral criteria for autoregressive (AR) expansions is of great practical importance for single parameter stationary time series. While this problem has been solved satisfactorily in recent years, the corresponding problem for SOSRF has only now come to the forefront.

The major difficulty with SOSRF is that there is no unique definition of "past" and "future". The definitions of a deterministic and purely nondeterministic random field can therefore be given separately for the horizontal, vertical and "south west" or quarter plane past.

An AR expansion is an expansion for the linear least squares predictor given in terms of past **observations** rather than in terms of an innovation sequence based on the past. The original solution of the Kolmogorov-Wiener theory is based on the latter and is less easily used in practiced than an AR expansion. A paper is being prepared on the work done to date.

3. A central limit theorem with random indices for stationary linear processes [3]

A central limit theorem with random indices is obtained for stationary linear process $X_t - \mu = \sum_{j=0}^{\infty} a_j \eta_{t-j}$, where $\{\eta_t\}$ are independent and identically distributed random variables with mean zero and finite variance and $\sum_{j=0}^{\infty} |a_j| < \infty$.

4. Limit theorems for sample covariances of stationary linear processes

with applications to sequential estimation [4]

The strong consistency and rate of convergence are established under optimal conditions for the asymptotic variance of the sample mean of a stationary linear process . Applications are made to the problem of sequential point and fixed width confidence interval estimation of the mean of a stationary linear process.

References

[1] J. Farshidi, Autoregressive expansion of the linear predictor for stationary stochastic processes, UNC Center for Stochastic Processes Technical Report No. 360, March 92

[2] J. Farshidi, G. Kallianpur and V. Mandrekar, Spectral characterization and autoregressive expansion of linear predictors for second order stationary random fields (SOSRF), Part I: Half planes, UNC Center for Stochastic Processes Technical Report No. 381, December 1992

[3] I. Fakhre-Zakeri and J. Farshidi, A central limit theorem with random indices for stationary linear processes, UNC Center for Stochastic Processes Technical Report No. 363, April 1992

[4] I. Fakhre-Zakeri and J. Farshidi, Limit theorems for sample covariances of stationary linear processes with applications to sequential estimation, UNC Center for Stochastic Processes Technical Report No. 380, September 1992

LARS HOLST

Professor Lars Holst of the Royal Institute of Technology, in Stockholm, visited the Center in January 1992. Professor Holst is an authority on Poisson approximations and interacted with faculty and visitors, also presenting a colloquium on approximation of Stein's Method.

CHRISTIAN HOUDRÉ

Dr. Christian Houdré of the Department of Mathematics of the University of Maryland, now at the Department of Statistics of Stanford University, visited the Center for two months. He worked primarily on the ramification of wavelets in stochastic processes [1,2,3], the latter being ongoing collaboration with S. Cambanis. He also worked on stable stochastic processes jointly with M. Hernández [4] and on variance inequalities for functions of Gaussian random variables jointly with A. Kagan [5].

1. Wavelets, probability and statistics: Some bridges [1]

The rôle of some wavelet methods in probability and statistics is illustrated via a sample of three problems: We show how properties of processes can be read off properties of their wavelet transform. We discuss how the missing data problem can be approached via frames of complex exponentials. We explain how wavelets can be used to span classes of admissible estimators in non-parametric function estimation. It is also the purpose of this paper to show that bridges can be crossed in the other direction. Random products of matrices determine the smoothness of compactly supported wavelets. Non stationary prediction theory gives new results on frames in Hilbert space.

2. Path reconstruction of processes from missing and irregular samples [2]

A criterion is provided for the reconstruction of the paths of non-stationary band-limited processes using irregularly spaced samples by means of an interpolation formula. Its rate of convergence is studied along with its truncation error. These results provide irregular sampling theorems for, say, deterministic signals corrupted by additive noise, and a potential solution to the missing data problem: interpolation from sparse or missing data can be achieved under a density condition. The analysis involves classical results on non-harmonic Fourier series as well as more recent results on frames and wavelets.

3. Wavelet transforms of random processes [3]

A study has been initiated of the properties of wavelet transforms of random processes whose sampled values appear as coefficients in the wavelet approximation of the process at a given resolution. A natural question is which properties of the process are inherited to its wavelet transform, and, conversely, which properties of the process can be read-off properties of its wavelet transform. For random processes with finite second moment, properties such as periodicity, stationarity, harmonizability, and self-similarity, are characterized by means of analogous properties of their wavelet transforms at some scale: The properties of the wavelet transform characterize the corresponding properties of the increments of the process of order equal to the order of regularity of the analyzing wavelet.

4. Disjointness results for some classes of stable processes [4]

The disjointness of two classes of stable stochastic processes: moving averages and Fourier transforms is discussed. Results on the incompatibility of these two representations date back to Urbanik (1964). Here we extend varous earlier disjointness results to encompass larger classes of processes, allowing e.g. the noise of a moving average process to be nonstationary and showing that all moving average processes are Fourier transforms in the summability sense.

5. Variance inequalities for functions of Gaussian variables [5]

When X is a standard Gaussian random variable and G an absolutely continuous function, the inequality $Var[G(X)] \leq E[G'(X)]^2$ was proved in Nash (1958) and later rediscovered in Brascamp and Lieb (1976) as a special case of a general inequality in Chernoff (1981). All the proofs are based on properties of the Gaussian density. By using the characteristic function rather than the density, generalizations with higher order derivatives are obtained. The method also establishes potentially useful connections with Karlin's total positivity.

References

[1] C. Houdré, Wavelets, probability and statistics: Some bridges. UNC Center for Stochastic Processes Technical Report No. 376, October 1992

[2] C. Houdré, Path reconstruction of processes from missing and irregular samples. UNC Center for Stochastic Processes Technical Report No. 359, Feb. 92. *Ann. Probability*, to appear.

[3] S. Cambanis and C. Houdré, Wavelet transforms of random processes, in preparation

[4] M. Hernández and C. Houdré, Disjointness results for some classes of stable processes, UNC Center for Stochastic Processes Technical Report No. 375, October 1992

[5] C. Houdré and A. Kagan, Variance inequalities for functions of Gaussian variables, UNC Center for Stochastic Processes Technical Report No. 374, October 1992

TAILEN HSING

Professor Tailen Hsing of the Statistics Department of Texas A & M University visited the Center for the 1990-91 academic year. In addition to his work reported in last year's annual report, in the following paper he extended and completed the work on the estimation of the spectral density of harmonizable stable processes of Cambanis and Masry (1984).

1. Limit theorems for stable processes with application to spectral density estimation [1]

It is shown that for a nearly stationary moving average α-stable process Y and for each fixed $0 < p < \infty$, a weighted average of $|Y(t)|^p$ over $[-T, T]$ has an asymptotically $(2 \wedge \frac{\alpha}{p})$-stable distribution as $T \to \infty$. This is a partial extension of the limit theorems considered in Davis (1983) and LePage, Woodroofe and Zinn (1981). Applications of the results are made in the context of spectral density estimation of a harmonizable α-stable process. The spectral density estimator is the smoothed version of the pth absolute power of the tapered Fourier transform proposed in Cambanis and Masry (1984) and proven consistent when $0 < p < \alpha/2$. Here its asymptotic distribution is derived and is shown to be normal when $0 < p \leq \alpha/2$ and (α/p)-stable when $\alpha/2 < p < \alpha$. Also the best possible rates of convergence are determined and show that the rate of convergence is faster for p in $(0, \alpha/2)$.

References

[1] T. Hsing, Limit theorems for stable processes with application to spectral density estimation, UNC Center for Stochastic Processes Technical Report No. 366, June 1992

HARRY HURD - ANDRZEJ RUSSEK

Dr. Harry Hurd continued the systematic study of non-stationary processes which are periodically correlated jointly with Dr. Andrzej Russek of the Polish Academy of Science, Sopot, who has been a visitor to the Center since January 1992.

1. Stepanov almost periodically correlated and almost periodically unitary processes [1]

We extend the structure and properties of almost periodically correlated (APC) and almost periodically unitary (APU) processes, which were defined in the sense of Bohr, to a larger class of processes for which the sense of almost periodicity is that of Stepanov. These processes are not necessarily continuous in quadratic mean, as are the Bohr APC and APU processes, but yet exhibit a sense of almost periodicity. For example, processes formed by amplitude modulation $f(t)X(t)$ or time-scale modulation $X(t+f(t))$ of a wide sense stationary process $X(t)$ by a Stepanov APU and APC. The principal results on APC and APU processes are extended to the new class. We extend Gladyshev's characterization of APC correlation functions to Stepanov APC processes and show that their correlation functions are completely represented by a Fourier series having a countable number of coefficient functions that are Fourier transforms of complex measures. We show that Stepanov APU processes are also Stepanov APC and are given by $X(t) = U(t)[P(t)]$ where $\{U(t),\ t \in \mathbf{R}\}$ is a strongly continuous group of unitary operators and $P(t)$ is a vector-valued Stepanov almost periodic function. As in the case of Bohr APU processes, the preceeding fact leads to representations of $X(t)$ based on the spectral theory for unitary operators and for Stepanov almost periodic functions.

2. Almost periodically correlated processes on LCA groups [2]

For an almost periodic covariance $R(t+\tau, t) = E\{X(t+\tau)\overline{X(t)}\}$ of a second order stochastic process $X(t)$ indexed by an LCA group G, we show that the means $a(\lambda, \tau) = M_t\{R(t+\tau, t)\overline{\lambda(t)}\}$ are Fourier transforms of signed measures with finite total variation. We examine conditions under which $X(t)$ or, more precisely, its correlation $R(t+\tau, t)$, has a countable set of spectral characteristic exponents (or frequencies). We also consider the problem of finding a stationarizing shift and exhibit a class of G-valued random variables θ such that $Y(t) = X(t+\theta)$ is stationary. Finally we characterize the almost periodically correlated processes among the strongly harmonizable ones.

References

[1] H. Hurd and A. Russek, Stepanov almost periodically correlated and almost periodically unitary processes, UNC Center for Stochastic Processes Technical Report No. 368, July 1992

[2] H. Hurd and A. Russek, Almost periodically correlated processes on LCA groups, UNC Center for Stochastic Processes Technical Report No. 369, July 1992

RAJEEVA L. KARANDIKAR

Professor Karandikar of the Indian Statistical Institute, Delhi, visited the Center for four months in 1992. In addition to the completion of work done jointly with A. Bhatt and G. Kallianpur [1] and partially described in the last report, he collaborated with V.G. Kulkarni of the Operations Research department on the study of a second-order fluid flow model [2] and with G. Kallianpur on nonlinear transformations of abstract Wiener measure [3].

1. On interacting systems of Hilbert space valued diffusions [1]

A nonlinear Hilbert space valued stochastic differential equation where L^{-1} (L being the generator of the evolution semigroup) is not nuclear is investigated in the paper. Under the assumption of nuclearity of L^{-1}, the existence of a unique solution lying in the Hilbert space H has been shown by Dawson in an early paper. When L^{-1} is not nuclear, a solution in most cases lies not in H but in a larger Hilbert, Banach or nuclear space. Part of the motivation of the present paper is to prove under suitable conditions that a unique strong solution can still be found to lie in the space H itself. Uniqueness of the weak solution is proved without moment assumptions on the initial random variable.

A second problem considered is the asymptotic behavior of the sequence of empirical measures determined by the solutions of an interacting system of H-valued diffusions. It is shown that the sequence converges in probability to the unique solution Λ_0 of the martingale problem posed by the corresponding McKean-Vlasov equation.

2. Second-order fluid flow model of a data-buffer in random environment [2]

This paper considers a stochastic model of a data-buffer in a telecommunication network. Let $X(t)$ be the buffer-content at time t. The $\{X(t),\ t \geq 0\}$ process depends on a finite state continuous time Markov process $\{Z(t),\ t \geq 0\}$ as follows: during the time-intervals when $Z(t)$ is in state i, $X(t)$ is a Brownian motion with drift μ_i, variance parameter σ_i^2 and a reflecting boundary at zero. This paper studies the steady state analysis of the bivariate process $\{(X(t), Z(t)),\ t \geq 0\}$ in terms of the eigenvalues and eigenvectors of a non-linear matrix system. Algorithms are developed to compute the steady state distributions as well as moments.

Numerical work is reported to show that the variance parameter has a dramatic effect on the buffer content process. Thus buffer sizing done with first order fluid flow models (with zero variance parameters) should be used with care.

3. Nonlinear transformations of the canonical Gauss measure on Hilbert space and absolute continuity [3]

The papers of R. Ramer (1974) and S. Kusuoka (1982) investigate conditions under which the probability measure induced by a nonlinear transformation on abstract Wiener space (γ, H, B) is absolutely continuous with respect to the abstract Wiener measure μ. These conditions reveal the importance of the underlying Hilbert space H but involve the space B in an essential way. The present paper gives conditions solely based on H and takes as its starting point a nonlinear transformation $T = I + F$ on H. New sufficient conditions for absolute continuity are given which do not seem easily comparable with those of Kusuoka or Ramer but are more general than those of Buckdahn (1991) and Enchev (1991). The Ramer-Itô integral occurring in the expression for the Radon-Nikodym derivative is studied in some detail and, in the general context of white noise theory, it is shown to be an anticipative stochastic

integral which, under a stronger condition on the weak Gateaux derivative of F, is directly related to the Ogawa integral.

References

[1] A.G. Bhatt, G. Kallianpur and R.L. Karandikar, On interacting systems of Hilbert space valued diffusions, UNC Center for Stochastic Processes Technical Report No. 373, Sept. 92

[2] R.L. Karandikar and V.G. Kulkarni, Second-order fluid flow model of a data-buffer in random environment, UNC Center for Stochastic Processes Technical Report No. 370, July 92

[3] G. Kallianpur and R.L. Karandikar, Nonlinear transformations of the canonical Gauss measure on Hilbert space and absolute continuity, in preparation

JEAN-CLAUDE MASSÉ

Professor Jean-Claude Massé of the Department of Mathematics and Statistics of the Unviersity of Laval visited the Center for six weeks and studied, jointly with C.A. León, the properties of the simplicial median of Oja with a view towards the study of medians of random processes.

1. La médiane simpliciale d'Oja: existence, unicité et stabilité [1]

Oja (1983) examined various ways of measuring location, scatter, skewness and kurtosis for multivariate distributions. Among other measures of location, he introduced a generalised median, the Oja median. We study three fundamental theoretical properties of that median: existence, uniqueness and consistency.

References

[1] C.A. León and J-C Massé, La médiane simpliciale d'Oja: existence, unicité et stabilité, UNC Center for Stochastic Processes Technical Report No. 362, March 1992

DITLEV MONRAD

Professor Ditlev Monrad of the Department of Statistics of the University of Illinois visited the Center for four months and worked on sample function properties of Lévy processes with local time, and of fractional Brownian motion and other locally nondeterministic Gaussian processes. The latter work was done jointly with H. Rootzén.

1. Some uniform dimension theorems for the sample functions of Lévy processes with local times [1]

When X is a real-valued, strictly stable Lévy process of index α, $1 < \alpha < 2$, we show that with probability one,

$$\dim X^{-1}(F) = 1 - \frac{1}{\alpha} + \frac{1}{\alpha} \dim (F),$$

simultaneously for all real Borel sets F. The result is also extended to general real-valued Lévy processes with local time.

2. Small values of fractional Brownian motion and locally nondeterministic Gaussian processes [2]

A centered Gaussian process $\{B_\alpha(t) : t \geq 0\}$ with covariance function proportional to $\frac{1}{2}\{|s|^\alpha + |t|^\alpha - |s-t|^\alpha\}$ for $\alpha \in (0,2)$ is called a fractional Brownian motion (fBm). Chung type laws of the iterated logarithm are proved for fBm's: for any sample path, there are arbitrarily large values of t for which $\{B_\alpha(s) : 0 \leq s \leq t\}$ is confined to the interval $\pm$ const. $t^{\alpha/2}(\log\log t)^{-\alpha/2}$, but this is not true for any narrower intervals. A corresponding result holds for small values of t. Let $M(t) = \max_{0 \leq s \leq t}|B_\alpha(s)|$. For the proof the following bounds, valid for small ϵ's, and constants $0 < c \lesssim C$ are found

$$e^{-Ct\epsilon^{-2/\alpha}} \leq P(M(t) \leq \epsilon) \leq e^{-ct\epsilon^{2/\alpha}},$$

for the probability that the process is flat. They hold for strongly locally nondeterministic Gaussian processes whose incremental variances over intervals of length h are roughly proportional to h^α.

References

[1] D. Monrad, Some uniform dimension theorems for the sample functions of Lévy processes with local times, UNC Center for Stochastic Processes Technical Report No. 386, January 1993

[2] D. Monrad and H. Rootzén, Small values of fractional Brownian motion and locally nondeterministic Gaussian processes, UNC Center for Stochastic Processes Technical Report No. 361, March 1992

JOHN P. NOLAN

Professor John Nolan of the Department of Mathematics and Statistics of the American University in Washington, DC, completed some work on multidimensional stable distributons which was substantially performed during a 1990 visit to the Center but was not included in the 1989-90 annual report.

1. Approximation of multidimensional stable densities [1]

Stable densities in two or more variables do not generally have explicit formula. One way of characterizing these distributions is by a spectral measure. Our main result shows that densities and probabilities can be uniformly approximated by approximating the spectral measure with a discrete spectral measure having a finite number of atoms. A concrete formula is given for the number of atoms needed and their weights, this can be used to numerically calculate multidimensional stable densities. Sample graphs of two dimensional stable densities with dependence are given.

References

[1] T. Byczkowski, J.P. Nolan and B. Rajput, Approximation of multidimensional stable densities, UNC Center for Stochastic Processes Technical Report No. 351, Oct. 1991

TOMMY NORBERG

Dr. Tommy Norberg of the University of Göteborg visited for a one month period in November 1991, primarily to collaborate with M.R. Leadbetter on problems in point process theory. Dr. Norberg is a foremost authority on the theory of random sets, which provide a useful alternative framework complementing those of point processes and random measures.

During the visit Dr. Norberg and M.R. Leadbetter worked together in three areas:

(a) Foundations for a non topological (or minimally topological) theory of point processes

(b) The potential use of random sets in applied areas - such as minefield modeling in defense applications

(c) Planning for a volume on point processes, random sets and random measures. This will describe the different structural frameworks, their relationships, the usefulness of each view and some of their applications.

Work has continued in these areas since the visit and a start has been made on the writing under (c).

JONNY OLSSON

Dr. Jonny Olsson (University of Lund, Sweden) was a junior visitor, supported from Swedish sources for November 1991. He collaborated with H. Rootzén on Markov random fields for vision.

1. Image Modeling [1]

Markov random field models for vision were developed jointly with J. Olsson, a junior short term visitor (noted above), describing the theory and application to peripheral vision assessment, and is summarized as follows.

Measurement of the patient's "seeing threshold" at different points in the visual field is an important diagnostic tool for glaucoma and other diseases. A Markov random field model is developed and used for efficient estimation of the thresholds and simultaneously for classification of the measured points as "normal" or "defective". The model allows for nonhomogeneous spatial dependence and nonsymmetric marginal distributions and has physically interpretable parameters. Maximum a posteriori threshold estimation of visual fields results in 13% - 31% reduction of mean square error (depending on the patient population) as compared to currently used procedures and in a fair agreement between true and estimated defect status.

Nonstandard features of the problem are: (i) the picture is small, (ii) there is a nonhomogeneous directional dependence, and (iii) thresholds are only measured indirectly, by binary responses to questions, where the probability of response depends on the threshold and the stimulus level.

References

[1] J. Olsson and H. Rootzén, An image model for quantal response analysis in perimetry, UNC Center for Stochastic Processes Technical Report No. 355, Nov. 1991

VASSILIS PAPANICOLAOU

Professor Papanicolaou of the Department of Mathematics of Duke University [and now at the Department of Mathematics of Wichita State University] was at the Center for two months in the summer of 1992. His interest in Feynman integrals led to the study of some problems of integration over Hilbert space and extensions involving multiparameter Gaussian processes of previous work by G. Kallianpur, D. Kannan and R.L. Karandikar (Analytic and sequential Feynman integrals on abstract Wiener and Hilbert spaces, and a Cameron-Martin formula, *Ann. Inst. H. Poincare*, **21**, 1985, 323-361).

1. Integration over Hilbert spaces: Examples inspired by the harmonic oscillator [1]

The research produced some examples of functional integrals over Hilbert spaces where the integrand is analogous to the one for the quantum mechanical harmonic oscillator. In one case the continuum limit of a sequence of coupled harmonic oscillators is considered.

References

[1] G. Kallianpur and V. Papanicolaou, Integration over Hilbert spaces: Examples inspired by the harmonic oscillator, UNC Center for Stochastic Processes Technical Report No. 367, July 92

ROLAND PERFEKT

Dr. Roland Perfekt (University of Lund, Sweden) was a junior visitor, supported from Swedish sources for the period September-October 1991. He collaborated with H. Rootzén on extremal properties of stationary Markov chains.

1. Extremal behaviour of stationary Markov chains with applications [1]

Extremal behaviour of real-valued, stationary Markov chains is studied under rather general assumptions. Conditions are obtained for convergence in distribution of multi-level exceedance point processes associated with suitable families of 'increasing levels'. Although applicable to general stationary sequences, these conditions are tailored for Markov chains and are seen to hold for a large class of chains. The extra assumptions needed are that the marginal distributions belong to the domain of attraction of some extreme value law together with rather weak conditions on the transition probabilities. Also, a complete convergence result is given. The results are applied to a discrete-time Lindley process, to an $AR(1)$ process with uniform margins and to solutions of a first order stochastic difference equation with random coefficients.

References

[1] R. Perfekt, Extremal behaviour of stationary Markov chains with applications, UNC Center for Stochastic Processes Technical Report No. 353, Nov. 1991

HOLGER ROOTZÉN

Professor Holger Rootzén (University of Lund, Sweden) spent 12 months as a senior visitor to the Center. In this period he also arranged shorter visits by two junior Swedish statisticians J. Olsson and R. Perfekt (supported from Swedish sources).

Professor Rootzén's activities were divided into five areas as follows.

1. Tail estimation for stationary sequences [1]

Holger Rootzén and M.R. Leadbetter collaborated in extending work reported previously (as CSP Report 292 with L. de Haan) on the estimation of parameters associated with high values of stochastic sequences. These include the "extremal index", the parameter of a regularly varying tail distribution, tail probabilities and quantiles, under dependence conditions such as strong mixing.

It is planned that this work will be reported in two papers (currently under preparation), dividing and extending that in the CSP Report 292.

2. Extremal properties of Markov chains [2]

Professor Rootzén worked with R. Perfekt on extremal properties of Markov chains. This is described in the research activity summary for R. Perfekt.

3. Fractional Brownian motion [3] (with D. Monrad)

Joint work was conducted on sample function properties of fractional Brownian motion and locally nondeterministic Gaussian processes. This work is described in CSP Tech Report No. 361, whose contents are summarized as follows:

A centered Gaussian process $\{B_\alpha(t) : t \geq 0\}$ with covariance function proportional to $\frac{1}{2}\{|s|^\alpha + |t|^\alpha - |s-t|^\alpha\}$ for $\alpha \in (0,2)$ is called a fractional Brownian motion (fBm). Chung type laws of the iterated logarithm are proved for fBm's: for any sample path, there are arbitrarily large values of t for which $\{B_\alpha(s) : 0 \leq s \leq t\}$ is confined to the interval $\pm$ const. $t^{\alpha/2}(\log\log t)^{-\alpha/2}$, but this is not true for any narrower intervals. A corresponding result holds for small values of t. Let $M(t) = \max_{0 \leq s \leq t}|B_\alpha(s)|$. For the proof the following bounds, valid for small ϵ's, and constants $0 < c \lesssim C$ are found

$$e^{-Ct\epsilon^{-2/\alpha}} \leq P(M(t) \leq \epsilon) \leq e^{-ct\epsilon^{2/\alpha}},$$

for the probability that the process is flat. They hold for strongly locally nondeterministic Gaussian processes whose incremental variances over intervals of length h are roughly proportional to h^α.

4. Image Modeling [4]

Markov random field models for vision were developed jointly with J. Olsson, a junior short term visitor (noted above), describing the theory and application to peripheral vision assessment, and is summarized as follows.

Measurement of the patient's "seeing threshold" at different points in the visual field is an important diagnostic tool for glaucoma and other diseases. A Markov random field model is developed and used for efficient estimation of the thresholds and simultaneously for classification of the measured points as "normal" or "defective". The model allows for nonhomogeneous spatial dependence and nonsymmetric marginal distributions and has physically interpretable parameters. Maximum a posteriori threshold estimation of visual fields results in 13% - 31% reduction of mean

square error (depending on the patient population) as compared to currently used procedures and in a fair agreement between true and estimated defect status.

Nonstandard features of the problem are: (i) the picture is small, (ii) there is a nonhomogeneous directional dependence, and (iii) thresholds are only measured indirectly, by binary responses to questions, where the probability of response depends on the threshold and the stimulus level.

5. Related statistical questions

Professor Rootzén conducted related statistical research in

(i) **Quantile estimation in a nonparametric component of variance framework with applications to vision problems** [5].

(ii) **Proportional hazard testing related to strength of materials** [6].

His results under (i) are summarized as follows:

Quantile estimators for a non-parametric components of variance situation are proposed and consistency and asymptotic normality is proved. Situations with different numbers of measurements for different subjects are considered. Measurements on separate subjects are assumed independent while measurements on the same subject have a fixed dependence. The estimators are obtained by inverting weighted empirical distribution functions. An "optimal" estimator is derived by choosing weights to minimize the variance of the weighted empirical distribution function. The resulting weights depend on unknown parameters. However, these weights may be estimated from data without affecting asymptotic performance. A simple estimator based on within subject averages is also investigated. Small sample properties are studied by simulation, and as an illustration the estimators are applied to normal limits for differential light sensitivity of the eye.

The work on proportional hazards (joint with A. Deis) provided a k-sample test for proportional hazards and is described in detail as follows:

A test for proportionality of the cumulative hazard functions in $k \geq 2$, possibly censored, samples is proposed. The test does not use dummy time-dependent covariates or partitions of the time axis. It extends a test of Wei (1984) from 2 to k samples, and for $k = 2$ gives an alternative approximation to the test probabilities. It is asymptotically correct and performed well in a small sample simulation study. The test is based on the maximum norm of the score process obtained from Cox' partial likelihood. The test probabilities are obtained by a "parametric bootstrap", i.e. by simulation from the asymptotic distribution, with an unknown variance function replaced by an estimate. The method is computationally demanding, but still within the capabilities of a standard personal computer. An important advantage is flexibility; by obvious simple changes the program can be used with any test statistic based on the score process. Some problems related to the size effect in the strength of materials are discussed, and the method is applied to a data set on the strengths of carbon fibers. It is also illustrated on two cancer studies considered by Wei.

References

[1] M.R. Leadbetter and H. Rootzén. Tail estimation for stationary sequences. in preparation

[2] R. Perfekt. Extremal properties of stationary Markov chains with applications. UNC Center for Stochastic Processes Technical Report No. 353, November 1991

[3] D. Monrad and H. Rootzén, Small values of fractional Brownian motion and locally nondeterministic Gaussian processes, UNC Center for Stochastic Processes Technical Report No. 361, March 1992

[4] J. Olsson and H. Rootzén, An image model for quantal response analysis in perimetry, UNC Center for Stochastic Processes Technical Report No. 355, Nov. 1991

[5] H. Rootzén, Quantile estimation in a nonparametric component of variance framework with applications to vision problems, UNC Center for Stochastic Processes Technical Report No. 385, Dec. 1992

[6] A. Deis and H. Rootzén, A k-sample test for proportional hazards with an application to the strength of materials, UNC Center for Stochastic Processes Technical Report No. 384, Dec. 1992

DONATAS SURGAILIS

Dr. Donatas Surgailis of the Institute of Mathematics and Informatics of the Lithuanian Academy of Sciences in Vilnius visited the Center for three months. He introduced a new rich class of stationary stable processes generalizing moving averages jointly with S. Cambanis, V. Mandrekar and J. Rosinski.

1. Generalized stable moving averages [1]

No explicit representation is known for all stationary non-Gaussian stable processes. The main two subclasses studied, which have explicit representations motivated by the Gaussian case, are the harmonizable processes, which are superpositions of harmonics with stable amplitudes, and the moving average processes, which are filtered white stable noise. While in the Gaussian case, the latter is a subclass of the former, in the non-Gaussian stable case the two classes are disjoint. The study of stable moving average processes is facilitated by the fact that their distribution is essentially (except for a translation and sign) determined by the filter function of the moving average. This has made it possible to study distributional properties of the process (mixing, ergodicity, self-similarity, Markov property, etc.) through the properties of the filter functions.

In this work the class of non-Gaussian stable moving average processes is expanded substantially by the introduction of an appropriate joint randomization of the filter function and of the stable noise, leading to stable generalized moving averages (GMA). The characterization of their distribution through their filter function and their mixing measure leads to a far reaching generalization of a theorem of Kanter (1972).

It is shown that stable GMA's contain sums of independent stable moving averages and that they are still disjoint from the harmonizable processes, but are closed under time invariant filters, and that they are mixing, so they have strong ergodic properties. They lead to a wealth of new examples of self-similar processes, beyond the linear fractional stable motions, and also of processes which are reflection positive, which is a useful weakening of the Markov property.

References

[1] S. Cambanis, V. Mandrekar, J. Rosinski and D. Surgailis, Generalized stable moving averages, UNC Center for Stochastic Processes Technical Report No. 365, April 1992

WEI WU

Professor Wei Wu of the Statistics Department of the University of Illinois visited the Center for one month and worked jointly with S. Cambanis and E. Carlstein on an extensive revision and generalization of a part of her Ph.D. dissertation research contained in the following paper.

1. Bootstrapping the sample mean for data with infinite variance [1]

When data comes from a distribution belonging to the domain of attraction of a stable law, Athreya (1987) showed that the bootstrapped sample mean has a random limiting distribution, implying that the naive bootstrap could fail in the heavy-tailed case. The goal here is to classify all possible limiting distributions of the bootstrapped sample mean when the sample comes from a distribution with infinite variance, allowing the broadest possible setting for the (nonrandom) scaling, the resample size, and the mode of convergence (in law). The limiting distributions turn out to be infinitely divisible with possibly random Lévy measure, depending on the resample size. An averaged-bootstrap algorithm is then introduced which eliminates any randomness in the limiting distribution. Finally, it is shown that (on the average) the limiting distribution of the bootstrapped sample mean is stable if and only if the sample is taken from a distribution in the domain of (partial) attraction of a stable law.

References

[1] W. Wu, E. Carlstein and S. Cambanis. Bootstrapping the sample mean for data with infinite variance, UNC Center for Stochastic Processes Technical Report No. 296, May 1990. Revision in preparation.

PH.D. STUDENTS

PH.D. DEGREES AWARDED

DAVID G. BALDWIN

Dr. Baldwin completed his Ph.D. degree working under the direction of G. Kallianpur. His thesis is described in item 1 below.

1. Topics in the theory of stochastic processes taking values in the dual of a countably Hilbertian nuclear space [1]

A theorem is given on the weak approximation of solutions to infinite dimensional stochastic differential equations. An example is given of the weak approximation of a spatial neuronal model with reversal potentials by a continuous diffusion taking values in the dual of a countaly Hilbertian nuclear space.

Lastly we give conditons for existence and uniqueness of global McKean-Vlasov equations. Results are extended to local McKean-Vlasov equations.

References

[1] D.G. Baldwin, Topics in the theory of stochastic processes taking values in the dual of a countably Hilbertian nuclear space, *Dissertation*.

Dr. Xiong completed his Ph.D. degree working under the direction of G. Kallianpur. His thesis is described in item 1 below. He has worked jointly with G. Kallianpur in developing various aspects of the theory of infinite dimensional stochastic differential equations listed in items 2 to 5 below.

1. Nuclear space valued stochastic differential equations driven by Poisson random measures [1]

This thesis is devoted primarily to the study of stochastic differential equations on duals of nuclear spaces driven by Poisson random measures. The existence of a weak solution is obtained by the Galerkin method and the uniqueness is established by implementing the Yamada-Watanabe argument in the present setup.

When the magnitudes of the driving terms are small enough and the Poisson streams occur frequently enough, it is proved that the stochastic differential equations mentioned above can be approximated by diffusion equations.

Finally, we consider a system of interacting stochastic differential equations driven by Poisson random measures. Let $(X_1^n(t), \ldots, X_n^n(t))$ be the solution of this system and consider the empirical measures

$$\zeta_n(\omega, B) \equiv \frac{1}{n} \sum_{j=1}^{n} \delta^{X_j^n(\cdot,\omega)}(B) \quad (n \geq 1).$$

It is proved that ζ_n converges in distribution to a non-random measure which is the unique solution of a McKean-Vlasov equation.

The above problems are motivated by applications in neurophysiology, in particular to the fluctuation of voltage potentials of spatially distributed neurons and to the study of asymptotic behavior of large systems of interacting neurons.

2. The existence and uniqueness of the solution of nuclear space-valued stochastic differential equations driven by Poisson random measures (with G. Hardy, S. Ramasubramanian and J. Xiong) [2]

In this paper, we study SDE's on duals of nuclear spaces driven by Poisson random measures. The existence of a weak solution is obtained by the Galerkin method. For uniqueness, a class of ℓ^2-valued processes which are called Good processes is introduced. An equivalence relation is established between SDE's driven by Poisson random measures and those by Good processes. The uniqueness is established by extending the Yamada-Watanabe argument to the SDE's driven by Good processes. This is an extension to discontinuous infinite dimensional SDE's of work done by G. Kallianpur, I. Mitoma and R. Wolpert for nuclear space valued diffusions [Stochastics, 29, 1-45, (1990)].

3. Stochastic differential equations in infinite dimensions: A brief survey and some new directions of research [3]

This is a brief survey of some recent work on nuclear space valued stochastic differential equations. The emphasis is on stochastic differential equations driven by Poisson random measures. An application of the evolution equation is made to stochastic models of environmental pollution. The asymptotic behavior of interacting systems of nuclear space valued, Poisson-driven SDE's is examined and a propagation of chaos result is presented. Some new directions of work are suggested.

4. Asymptotic behavior of a system of interacting stochastic differential equations driven by Poisson random measures [4]

In this paper, we study a system of interacting stochastic differential equations taking values in nuclear spaces and driven by Poisson random measures. We also consider the McKean-Vlasov equation associated with the system. We show that under suitable conditions the system has a unique solution and the sequence of its empirical distributions converges to the solution of the McKean-Vlasov equation when the size of the system tends to infinity. The results are applied to the voltage potentials of a large system of neurons and a law of large numbers for the empirical measure is obtained.

5. Stochastic models of environmental pollution [5]

In this paper, we consider several stochastic models arising from environmental problems. First, we study the pollution in a domain where undesired chemicals are deposited at random times and locations according to Poisson streams. Incorporated with drift and dispersion, the chemical concentration can be modeled by a linear stochastic partial differential equation (SPDE) which is solved by applying a general result. Various properties, especially the limit behavior of the pollution process, are discussed. Secondly, we consider the pollution problem when a tolerance level is imposed. The chemical concentration can still be modeled by a SPDE but is no longer linear. Its properties are investigated in this paper. Finally, the linear filtering is considered based on the data of several observation stations.

References

[1] J. Xiong, Nuclear space valued stochastic differential equations dirven by Poisson random measures, UNC Center for Stochastic Processes Technical Report No. 364, April 1992, *Dissertation*

[2] G. Kallianpur, J. Xiong, G. Hardy and S. Ramasubramanian, The existence and uniqueness of solutions of nuclear space-valued stochastic differential equations driven by Poisson random measures, UNC Center for Stochastic Processes Technical Report No. 348, Sept. 91

[3] G. Kallianpur and J. Xiong, Stochastic differential equations in infinite dimensions: A brief survey and some new directions of research. Centerf for Stochastic Processes Technical Report No. 372, Sept. 92

[4] G. Kallianpur and J. Xiong, Asymptotic behavior of a system of interacting stochastic differential equations driven by Poisson random measures, in preparation

[5] G. Kallianpur and J. Xiong, Stochastic models of environmental pollution, in preparation

DISSERTATION IN PROGRESS

A. BUDHIRAJA

A contribution to the theory of McShane stochastic integrals

Under the direction of G. Kallianpur, Amarjit Budhiraja is developing the theory of McShane stochastic integrals. The topic contains results that are extensions of some of the results obtained by G.W. Johnson and G. Kallianpur and reported in an earlier annual scientific report.

JOURNAL PUBLICATIONS

1. J.M.P. Albin, On the general law of iterated logarithm with application to Gaussian processes in $\mathbf{R}^n$ and Hilbert space and to stable processes, *Stochastic Proc. Appl.*, **41**, 1992, 1-31.

2. F. Avram and M. Taqqu, Weak convergence of sums of moving averages in the α-stable domain of attraction, *Ann. Probability*, **20**, 1992, 483-503.

3. K. Benhenni and S. Cambanis, Sampling designs for estimating integrals of stochastic processes, *Ann. Statist.*, **20**, 1992, 161-194.

4. K. Benhenni and S. Cambanis, Sampling designs for estimating integrals of stochastic processes using quadratic mean derivatives, *Approximation Theory*, G. Anastassiou, ed., M. Dekker, 93-123.

5. R.C. Bradley, On the spectral density and asymptotic normality of weakly dependent random fields, *J. Theor. Probab.*, **5**, 1992, 355-373.

6. S. Cambanis, Random filters which preserve the normality of non-stationary random inputs, *Nonstationary Stochastic Processes and their Applications*, A.G. Miamee ed., World Scientific, 1992, 219-237.

7. S. Cambanis, M. Maejima and G. Samorodnitsky, Characterizations of one-sided linear fractional Lévy motions, *Stochastic Proc. Appls.*, **42**, 1992, 91-110.

8. S. Cambanis and E. Masry, Trapezoidal stratified Monte Carlo integration. *SIAM J. Numer. Anal.*, **29**, 1992, 284-301.

9. S. Cambanis and W. Wu, Multiple regression on stable vectors, *J. Multivariate Anal.*, **41**, 1992, 243-272.

10. X. Fernique, Sur les espaces de Fréchet ne contenant pas c_0, *Studia Math.*, **101**, 1992, 299-309.

11. C. Houdré, A note on the dilation of second order processes, *Nonstationary Stochastic Processes and their Applications*, A.G. Miamee, ed., World Scientific, 1992, 238-242.

12. T. Hsing and R.J. Carroll, An asymptotic theory for sliced inverse regression, *Ann. Statist.*, **20**, 1992, 1040-1061.

13. H.L. Hurd, Almost periodically unitary stochastic processes, *Stochastic Proc. Appls.*, **43**, 1992, 99-113.

14. H. Hurd and G. Kallianpur, Periodically correlated and periodically unitary processes and their relationship to $L^2[0,T]$-valued stationary sequences, *Nonstationary Stochastic Processes and their Applications*, A.G. Miamee ed., World Scientific, 1992, 256-287.

15. H.L. Hurd and J. Leskow, Estimation of the Fourier coefficient functions and their spectral densities for ϕ-mixing almost periodically correlated processes, *Stat. Probab. Letters*, 14, 1992, 299-306.

16. M. Iizuka and Y. Ogura, Convergence of one-dimensional diffusion processes to a jump process related to population genetics, *J. Math. Biology*, **29**, 1991, 671-687.

17. H.M. Itô, Y. Ogura and M. Tomisaki, Stretched-exponential decay laws of general defect diffusion models, *J. Statist. Physics*, **66**, 1992, 563-582.

18. G.W. Johnson and G. Kallianpur, The analytic Feynman integral of the natural extension of pth homogeneous chaos, *Rend. Circ. Mat. Palermo*, Ser. II, **28**, 1992, 181-199.

19. O. Kallenberg, From optional skipping to random time change - on some recent advances in exchangeability theory, *Theory Probab. Appl.*, **37**, 1992, 64-74. (In Russian)

20. O. Kallenberg, Some time change representations of stable integrals, via predictable transformation of local martingales, *Stochastic Proc. Appl.*, **40**, 1992, 199-223.

21. O. Kallenberg, Symmetries on random arrays and set-indexed processes, *J. Theor. Probab.*, **5**, 1992, 727-765.

22. G. Kallianpur, A line grid method in areal sampling and its connection with some early work of H. Robbins, *Amer. J. Math. Manag. Sci.*, **11**, 1991, 40-53.

23. G. Kallianpur, Traces, natural extensions and Feynman distributions, *Gaussian Random Fields*, K. Itô and T. Hida, eds., World Scientific, 1991, 14-27.

24. G. Kallianpur and I. Mitoma, A Segal-Langevin-type stochastic differential equation on a space of generalized functionals, *Canadian J. Math.*, **44**, 1992, 524-552.

25. G. Kallianpur and V. Perez-Abreu, The Skorohod integral and the derivative operator of functionals of a cylindrical Brownian motion, *Appl. Math. Optimization*, **25**, 1992, 11-29.

26. G. Kallianpur and R. Selukar, Parameter estimation in linear filtering, *J. Multivariate Anal.*, **39**, 1991, 284-304.

27. G. Kallianpur and J. Xiong, A nuclear-space-valued stochastic differential equation driven by Poisson random measures, *Stochastic PDE's and their Applications*, B.L. Rozovskii and R.B. Sowers, eds., Lecture Notes in Control and Information Sciences No. 176, Springer, 1992, 135-143.

28. T. Koski, A nonlinear autoregression in the theory of signal compression, *Ann. Acad. Sci. Fenn.*, Ser. A.I. Math., **17**, 1992, 51-64.

29. T. Koski and S. Cambanis, On the statistics of the error in predictive coding for stationary Ornstein-Uhlenbeck-processes, *IEEE Trans. Information Theor.*, **38**, 1992, 1029-1040.

30. H.L. Koul, M-estimators in linear models with long range dependent errors, *Stat. Prob. Letters*, **14**, 1992, 153-164.

31. M.R. Leadbetter and H. Rootzén, On central limit theory for families of strongly mixing additive random functions, *Stochastic Processes, A Festschrift in Honor of Gopinath Kallianpur*, Springer, 1992, 211-224.

32. M. Marques and S. Cambanis, Dichotomies for certain product measures and stable processes, *Probab. Math. Stat.*, **12**, 1991, 271-289.

33. J. Mijnheer, U-statistics and double stable integrals, *Selected Proceedings of the Sheffield Symposium on Applied Probability*, I.V. Basawa and R.L. Taylor eds., IMS Lecture Notes - Monograph Series, Vol. 18, 1992, 256-269.

34. I. Rychlik, The two-barrier problem for continuously differentiable processes, *Adv. Appl. Probability*, **24**, 1992, 71-94.

Accepted for Publication

35. J.M.P. Albin, On the upper and lower classes for stationary Gaussian stochastic processes, *Ann. Probability*, to appear.

36. R.C. Bradley, An addendum to "A limitation of Markov representation for stationary processes", *Stochastic Proc. Appl.*, to appear.

37. R.C. Bradley, Equivalent mixing conditions for random fields, *Ann. Probability*, to appear.

38. R.C. Bradley, dSome examples of mixing random fields, *Rocky Mount. J. Math.*, to appear.

39. T. Byczkowski, J.P. Nolan and B. Rajput, Approximation of multidimensional stable densities, *J. Multivariate Anal.*, 1993, to appear.

40. S. Cambanis and C. Houdré, Stable processes: Moving averages versus Fourier transforms, *Probab. Theory Rel. Fields*, to appear.

41. T.S. Chiang, G. Kallianpur and P. Sundar, Propagation of chaos for systems of interacting neurons, *Proc. Trento Conf. on Stochastic Partial Differential Equations*, G. Da Prato et al., eds., Springer, 1992, to appear.

42. D. Daley and T. Rolski, Finiteness of waiting-time moments in general stationary single-server queues, *Ann. Appl. Probab.* 1992, to appear.

43. C. Houdré, On the spectral SLLN and pointwise ergodic theorem in L^α, *Ann. Probability*, **20**, 1992, to appear.

44. C. Houdré, Path reconstruction of processes from missing and irregular samples. *Ann. Probability*, to appear.

45. T. Hsing, On some estimates based on sample behavior near high level excursions, *Probab. Theory Rel. Fields*, to appear.

46. G.W. Johnson and G. Kallianpur, Homogeneous chaos, p-forms, scaling and the Feynman integral, *Trans. Amer. Math. Soc.*, to appear.

47. G. Kallianpur, Stochastic differential equation models for spatially distributed neurons and propagation of chaos for interacting systems, *J. Math. Biol.*, to appear.

48. G. Kallianpur and R. Selukar, Estimation of Hilbert space valued parameters by the method of sieves, *Current Issues in Statistics and Probability*, J.K. Ghosh et al., eds., Wiley, 1992, to appear.

49. M. Maejima and Y. Morita, Trimmed sums of mixing triangular arrays with stationary rows, *Yokohama Math. J.*, **40**, 1992, to appear.

50. J. Rosinski and G. Samorodnitsky, Distribusions of subadditive functionals of sample paths of infinitely divisible processes, *Ann. Probab.*, to appear.

51. G. Samorodnitsky, Integrability of stable processes, *Probability Math. Statist.*, to appear.

52. Y.C. Su and S. Cambanis, Sampling designs for estimation of a random process, *Stochastic Proc. Appl.*, 1993, to appear.

CENTER FOR STOCHASTIC PROCESSES
TECHNICAL REPORTS

[344] G. Kallianpur and J. Xiong, A nuclear-space-valued stochastic differential equation driven by Poisson random measures, Sept. 91. *Stochastic PDE's and their Applications*, B.L. Rozovskii and R.B. Sowers, eds., Lecture Notes in Control and Information Sciences No. 176, Springer, 1992, 135-143.

[345] O. Kallenberg, Symmetries on random arrays and set-indexed processes, Sept. 91. *J. Theor. Probab.*, **5**, 1992, 727-765.

[346] S. Cambanis, A.T. Lawniczak, K. Podgorski and A. Weron, Ergodicity and mixing of symmetric infinitely divisible processes, Sept. 91.

[347] Y.C. Su and S. Cambanis, Sampling designs for estimation of a random process, Sept. 91. *Stochastic Proc. Appl.*, 1993, to appear.

[348] G. Hardy, G. Kallianpur, S. Ramasubramanian and J. Xiong, The existence and uniqueness of solutions of nuclear space valued stochatic differential equations driven by Poisson random measures, June 92.

[349] H. Hurd and V. Mandrekar, Spectral theory of periodically and quasi-periodically stationary $S\alpha S$-sequences, Sept. 91.

[350] T. Hsing and M.R. Leadbetter, On the excursion random measure of stationary processes, Oct. 91.

[351] T. Byczkowski, J.P. Nolan and B. Rajput, Approximation of multidimensional stable densities, Oct. 91. *J. Multivariate Anal.*, 1993, to appear.

[352] S. Cambanis and E. Masry, Wavelet approximation of deterministic and random signals: convergence properties and rates, Nov. 91.

[353] R. Perfekt, Extremal behaviour of stationary Markov chains with applications, Nov. 91.

[354] S. Cambanis, Random filters which preserve the normality of non-stationary random inputs, Nov. 91. *Nonstationary Stochastic Processes and their Applications*, A.G. Miamee ed., World Scientific, 1992, 219-237.

[355] J. Olsson and H. Rootzén, An image model for quantal response analysis in perimetry, Nov. 91.

[356] Y.-C. Su, Sampling designs for estimation of regression coefficients and of a random process, Dec. 91. *Dissertation.*

[357] G. Kallianpur, Stochastic differential equation models for spatially distributed neurons and propagation of chaos for interacting systems, Dec. 91. *J. Math. Biol.*, to appear.

[358] H. Hurd and G. Kallianpur, Periodically correlated and periodically unitary processes and their relationship to $L^2[0,T]$-valued stationary sequences, Feb. 92. *Nonstationary Stochastic Processes and their Applications*, A.G. Miamee ed., World Scientific, 1992, 256-287.

[359] C. Houdré, Path reconstruction of processes from missing and irregular samples, Feb. 92. *Ann. Probability*, to appear.

[360] J. Farshidi, Autoregressive expansion of the linear predictor for stationary stochastic processes, Mar. 92.

[361] D. Monrad and H. Rootzén, Small values of fractional Brownian motion and locally nondeterministic Gaussian processes, Mar. 92.

[362] C.A. León and J-C Massé, La médiane simpliciale d'Oja: existence, unicité et stabilité , Mar. 92.

[363] I. Fakhre-Zakeri and J. Farshidi, A central limit theorem with random indices for stationary linear processes, Apr. 92.

[364] J. Xiong, Nuclear space-valued stochastic differential equations driven by Poisson random measures, Apr. 92. *Dissertation*

[365] D. Surgailis, J. Rosinski, V. Mandrekar and S. Cambanis, Stable generalized moving averages, Apr. 92.

[366] T. Hsing, Limit theorems for stable processes with application to spectral density estimation, June 92.

[367] G. Kallianpur and V.G. Papanicolaou, Integration over Hilbert spaces: examples inspired by the harmonic oscillator, July 92.

[368] H. Hurd and A. Russek, Stepanov almost periodically correlated and almost periodically unitary processes, July 92.

[369] H. Hurd and A. Russek, Almost periodically correlated processes on LCA groups, July 92.

[370] R.L. Karandikar and V.G. Kulkarni, Second-order fluid flow model of a databuffer in random environment, July 92.

[371] R. Cheng, Outer factorization of operator valued weight functions on the torus, July 92.

[372] G. Kallianpur and J. Xiong, Stochastic differential equations in infinite dimensions: A brief survey and some new directions, Sept. 92.

[373] A.G. Bhatt, G. Kallianpur and R.L. Karandikar, On interacting systems of Hilbert space valued diffusions, Sept. 92.

[374] C. Houdré and A. Kagan, Variance inequalities for functions of Gaussian variables, Oct. 92.

[375] M. Hernández and C. Houdré, Disjointness results for some classes of stable processes, Oct. 92.

[376] C. Houdré, Wavelets, probability and statistics: some bridges, Oct. 92.

[377] S. Nandagopalan, M.R. Leadbetter and J. Hüsler, Limit theorems for nonstationary strongly mixing vector random measures, Nov. 92.

[378] R. Lund, A dam with seasonal input, Nov. 92.

[379] R. Cheng, Operator valued functions of several variables: Factorization and invariant subspaces, Nov. 92.

[380] I. Fakhre-Zakeri and J. Farshidi, Limit theorems for sample covariances of stationary linear processes with applications to sequential estimation, Sept. 92.

[381] J. Farshidi, G. Kallianpur and V. Mandrekar, Spectral characterization and autoregressive expansion of linear predictors for second ordered stationary random fields, Part 1: half planes, Dec. 92.

[382] J. Farshidi, Spectral characterization and prediction of L^p-representable stochastic processes, and some related extremal problems in L^p-spaces, $(0 < p < \infty)$, Dec. 92.

[383] S. Cambanis and I. Fakhre-Zakeri, On prediction of heavy-tailed autoregressive sequences: Regression versus best linear prediction, Dec. 92.

[384] A. Deis and H. Rootzén, A k-sample test for proportional hazards with an application to the strength of materials, Dec. 92.

[385] H. Rootzén, Quantile estimation in a nonparametric component of variance framework with applications to vision problems, Dec. 92.

[386] D. Monrad, Some uniform dimension theorems for the sample functions of Lévy processes with local times, Jan. 93.

IN PREPARATION

K. Benheni and S. Cambanis, The effect of quantization on the performance of sampling designs, in preparation.

K. Benhenni, S. Cambanis and Y.C. Su, Sampling designs for regression coefficient estimation with correlated errors, in preparation.

S. Cambanis and C. Houdré, Wavelet transforms of random processes, in preparation.

G. Kallianpur and R.L. Karandikar, Nonlinear transformations of the canonical Gauss measure on Hilbert space and absolute continuity, in preparation.

G. Kallianpur and J. Xiong, Asymptotic behavior of a system of interacting stochastic differential equations driven by Poisson random measures, in preparation.

G. Kallianpur and J. Xiong, Stochastic models of environmental pollution, in preparation.

M.R. Leadbetter and H. Rootzén, Tail estimation for stationary sequences, in preparation.

Seminars

Sept. 11	J. Stasheff, UNC - Chapel Hill: *Taking the measure of string field theory*
Sept. 19	Vasilis G. Papanicolaou, Duke University: *Multidimensional Schrödinger operators with almost periodic potentials*
Sept. 25	D. Kölzow, University of Erlangen-Nürnberg: *Integral transform associated with fractional Brownian motion*
Oct. 2	C. Houdré, University of Maryland: *Path reconstruction of processes from irregular samples*
Oct. 9	K. Petersen, UNC - Chapel Hill: *Random sampling of stationary processes. Including a disucssuion of the divergent views of Birkhoff, von Neumann, and Wiener on convergence*
Oct. 11	R.E. Kalman, ETH, Zurich and University of Florida: *Identification of linear relations in noise*
Oct. 16	V. Wihstutz, UNC - Charlotte: *Stochastic averaging and stabilization by random vibration*
Oct. 18	Y. Kifer, Hebrew University, Jerusalem: *Averaging in dynamical systems and large deviations*
Oct. 23	B. Rozovskii, University of Southern California: *Stochastic partial differential equations and intermittency*
Oct. 30	D. Monrad, University of Illinois and Center for Stochastic Processes: *Some uniform dimension theorems for the sample functions of stable Lévy processes with local times*
Nov. 6	G. Lawler, Duke University: *Nearest neighbor cluster models*
Nov. 13	R. Andersen, UNC - Charlotte: *Long run average cost for an optimal replacement model and an optimal maintenance-replacement model in reliability*
Nov. 20	J.-C. Massé, Laval University and Center for Stochastic Processes: *On Gaussian reciprocal processes*
Nov. 21	T. Norberg, University of Göteborg and Center for Stochastic Processes: *On weak lumpability for discrete time finite state space Markov chains*
Nov. 26	H. Hurd: *Periodically and almost periodically correlated random processes*

Dec. 4	J. Rissanen, IBM Research Center, San Jose, CA: *"Universal" modeling and prediction of time series*
Jan. 22	J.A. Cima, UNC - Chapel Hill: *Wavelets: A theorem of Mallat*
Feb. 5	J.A. Cima, UNC - Chapel Hill: *Wavelets: A theorem of Daubechies*
Feb. 12	S.I. Resnick, Cornell University: *Convergence of scaled random samples in* $\mathbf{R}^d$
Feb. 19	H. Rootzén, University of Lund and Center for Stochastic Processes: *What is the probability that a Gaussian path is flat?*
Feb. 26	D. Surgailis, Lithuanian Academy of Sciences, Vilnius, and Center for Stochastic Processes: *Asymptotics of random solutions of the Burgers equation*
Mar. 11	I. Fakhre-Zakeri, University of Maryland and Center for Stochastic Processes: *Models of Empirical-Bayes type for software reliability: Identifiability and applications to optimal stopping of software testing*
Mar. 16	B.V. Rao, Indian Statistical Institute and Indiana University: *The dynamics of quadratic maps under random iteration*
Mar. 25	A. Russek, Polish Academy of Science, Sopot: *Regularity properties of the conditional expectation in nonlinear white noise filtering*
Apr. 2	I.M. Sonin, UNC-Charlotte: *The asymptotic behavior of finite nonhomogeneous Markov chains*
Apr. 9	J.-A. Yan, Institute of Applied Mathematics, Beijing: *White noise calculus in terms of formal series expansions*
Apr. 15	M. Scarsini, Universita D'Annunzio, Pescara, Italy and Duke University: *Some results on the comparison of stochastic processe*
Apr. 29	P. Hitczenko, North Carolina State University: *On domination inequality for certain martingale transforms*
Apr.-May	R.L. Karandikar, ISI and Center for Stochastic Processes: *A short course on Markov processes*
May 13	M.G. Nadkarni, University of Bombay and McGill University: *Some spectral questions in ergodic theory*
May 15	R.M. Gray, Stanford University: *Image compression and vector quantization: Clustering and classification trees*
May 20	R.L. Karandikar, Indian Statistical Institute and Center for Stochas-

tic Processes: *Invariant measures and evolution equations for Markov processes characterized via martingale problems*

May 27	T. Zak, Technical University, Wroclaw, Poland: *Some new results on Gaussian measures: Strictness of Anderson's inequality and isoperimetric properties of strips*
June 3	M. Burnashev, Russian Academy of Sciences, Moscow: *On a search and related large deviations problems*
June 5	A.H. Korezlioglu, Institute of Telecommunications, Paris: *Product form approximations of finite capacity networks*
June 10	R. Chaganty, Old Dominion University, Norfolk: *Large deviations for the bootstrap distrubutions*
June 17	I. Fakhre-Zakeri, University of Maryland and Center for Stochastic Processes: *A central limit theorem with random indices for stationary linear processes with applications*
June 24	C. Houdré, University of Maryland and Center for Stochastic Processes: *Variance inequalities for functions of Gaussian variables*
July 1	R.L. Karandikar, Indian Statistical Institute, Delhi, and Center for Stochastic Processes: *Stochastic differential equations with values in a Hilbert space and propogation of chaos*
July 8	J. Leskow, University of California, Santa Barbara: *Inference for repeatable events*
July 15	J. Farshidi, Center for Stochastic Processes: *Spectral characterization and autoregressive expansion of horizontal and vertical linear predictions for stationary second order random fields*
July 22	R. Cheng, University of Louisville and Center for Stochastic Processes: *A Wold-type decomposition for second-order stationary random fields*
Aug. 6	I. Weissman, Technion, Israel: *The indices of the largest observations among n independent ones*

PROFESSIONAL PERSONNEL

Faculty Investigators

S. Cambanis
G. Kallianpur
M.R. Leadbetter

Visitors

R. Cheng, University of Louisville	June - July 92
I. Fakhre-Zakeri, University of Maryland	January - December 92
J. Farshidi, Michigan State University	October 91 - December 92
L. Holst, University of Stockholm	January 92
C. Houdre, University of Maryland	15 May - 15 July 92
H. Hurd	entire period
R.L. Karandikar, Indian Statistical Institute, New Delhi	April - July 92
J.-C. Masse, University of Laval	mid October - November 92
D. Monrad, University of Illinois	September - December 91
T. Norberg, University of Göteborg, Sweden	November 91
J. Olsson, Lund University, Sweden	November 91
V. Papanicolaou, Duke University	May - June 92
R. Perfekt, Lund University, Sweden	September - October 91
H. Rootzen, Lund University, Sweden	September 91 - July 92
A. Russek, Polish Academy of Science, Sopot	January - December 92
D. Surgailis, Lithuanian Academy of Science, Vilnius	Sept. - Oct. 91, February 92
W. Wu, University of Illinois	August 92

Graduate Students

D. Baldwin
A. Budhiraja
J. Xiong

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Faculty Investigator: S. Cambanis

Visitors: R. Cheng, I. Fakhre-Zakeri, J. Farshidi, C. Houdré, D. Monrad, T. Norberg, H. Rootzén, D. Surgailis, W. Wu.

Graduate Student: J. Xiong

ARMY RESEARCH OFFICE

Faculty Investigator: G. Kallianpur

Visitors: R.L. Karandikar, V. Papanicolaou, D. Surgailis

Graduate Student: J. Xiong